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# PROTON DIFFRACTION DISSOCIATION AND UNITARITY

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**Abstract**

It is shown that the dipole pomeron model of single diffraction dissociation – contrary to the case of the supercritical pomeron – is compatible with the inequality $\sigma^{SD} \leq \sigma^{tot}$, imposed by unitarity, provided the triple pomeron coupling satisfies certain conditions. With the adopted approximation the model considered as a parcticular solution of the triple pomeron decoupling problem. Explicit forms of such a coupling and a qualitative comparison with the experimental data on single DD are presented. The modified factorization properties of the model are also discussed.

# 1 Introduction

The renewed interest in hadrons diffraction dissociation (DD), observed recently has its origin in a different class of events, namely diffractive deep inelastic scattering, in which the notion of "pomeron-proton" scattering, or "pomeron flux" is introduced in terms of the single DD cross section (see e.g. [1] - [3] and [4] for a recent presentation of the problem).

Let us remind that the differential cross section for single DD in the triple-Regge kinematical limit, $M^2 \gg s_0, s/M^2 \gg 1$ (Fig.1) with account for only one, leading (pomeron) trajectory in each channel is

$$M^2 \frac{d\sigma}{dtdM^2} = \frac{\beta_{h\mathbb{P}}(0)\beta^2_{h\mathbb{P}}(t)G_{3\mathbb{P}}(t)}{16\pi}\left(\frac{M^2}{s_0}\right)^{\alpha(0)-1}\left(s/M^2\right)^{2\alpha(t)-2}$$

$$= f_{\mathbb{P}/p}(x_{\mathbb{P}}, t)\sigma^{tot}_{\mathbb{P}p}(M^2, t), \qquad (1)$$

where

$$\sigma^{tot}_{\mathbb{P}h}(M^2, t) = \beta_{h\mathbb{P}}(0)G_{3\mathbb{P}}(t)\left(\frac{M^2}{s_0}\right)^{\alpha(0)-1}$$

is the total cross section of the fictious "pomeron-hadron" scattering, and

$$f_{\mathbb{P}/h}(x_{\mathbb{P}}, t) = \frac{1}{16\pi}\beta^2_{h\mathbb{P}}(t)x_{\mathbb{P}}^{1-2\alpha(t)}, \qquad x_{\mathbb{P}} = M^2/s$$

is the so-called "pomeron flux", the probability that the hadron emmits a pomeron.

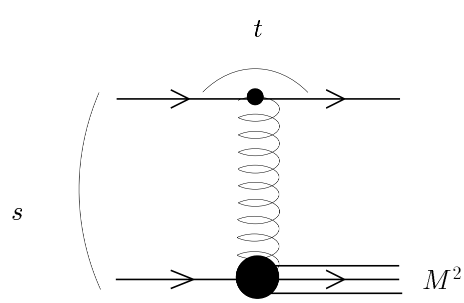


Fig.1. Hadron diffraction dissociation.

There is an old but still topical problem, namely the problem of pomeron decoupling (see e.g.[9]). If the pomeron is a simple pole with a linear trajectory of unite intercept, then $\sigma^{tot}(s) \to const$ at $s \to \infty$. At the same time, if $G_{3\mathbb{P}} \neq 0$ at $t = 0$ then it follows from (1) that $\sigma^{SD} \sim lnlns$ at $s \to \infty$. Moreover, the cross-sections of more complicated processes (such as double diffraction, central diffraction production and so on) rise with energy even faster than $\sigma^{SD}$, thus violating unitarity. It was suggested that the pomerons are (in the case of simple poles) decoupled in a $3\mathbb{P}$-vertex at $t = 0$ and consequently $d\sigma/dtdM^2$ vanishes when $t \to 0$. However later this was shown (see the review [10]) to contradict the experimental data. Thus a serious inconsistency between the theory and experiment emerged. We hope that the problem can be solved in the model under consideration. Here for simplicity we consider only single DD. A complete treatment of all diffractive contributions will be give elsewhere.

In most of the phenomenological applications a "supercritical" pomeron i.e. one with the intercept beyond unity, $\alpha(0) = 1 + \delta,\ \delta > 0$ is used. The value of $\delta$ in reactions in question (involving on mass shell protons) is about 0.1, corresponding to a "soft" pomeron according to the

widely used terminology. This model is attractive for its simplicity: as shown by A.Donnachie and P.Landshoff [5] , a single power term mimics, in a limited energy range the contribution of the pomeron that otherwise is a very complicated object. Due to the smallness of the parameter $\delta$, the total cross section with such a pomeron does not violate in a huge energy range the Froissart bound, following from unitarity. (Unitarity bounds become more problematic however in the case of diffraction dissociation to be discussed in this paper.) A single pomeron term has also the virtue of respecting exactly factorization, crucial in most of the studies of diffractive deep inelastic scattering.

Consider now proton single DD, to be denoted in what follows as SD. Prior to the highest energy Tevatron measurements, data on the integrated DD cross section were well fitted by a supercritical pomeron with $\delta \approx 0.1$, as discussed above. Such a behaviour, however conflicts with the data at higher energies, that lie well below the relevant extrapolations (see Fig. 2). Moreover, the DD cross section $\sim s^{2\delta}$ rises faster than the total cross section $\sim s^{\delta}$ and overshoots the latter already in the energy range of the present accelerators.

In a recent paper [4] Goulianos suggested a piece-wise "unitarization" by introducing a threshold in energy with different normalizations for the pomeron flux below and above the threshold. Consequently, the DD cross section gets a "knee" near that threshold with an abrupt change in the rate of increase.

Although data on DD can be fitted in this way, one can hardly imagine the dynamics to be discontinuous. Moreover, the inequality $\sigma^{SD}(\sim s^{2\delta}) \leq \sigma^{tot}(\sim s^{\delta})$ can not be satisfied with a supercritical pomeron since integration in $t$ (see below) introduces only logarithmic factors in $t$. Attempts to solve this problem by summing up an infinite series of unitarity corrections were undertaking in Refs. [6, 7]. However neither the unitarization method nor the final result are yet conclusive. Below we show that the inequality $\sigma^{SD}(\sim s^{2\delta}) \leq \sigma^{tot}(\sim s^{\delta})$ can be satisfied for a dipole pomeron. Interestingly, the application of this bound constrains the form of the triple pomeron vertex. We present a simple example of such a solution and show also that the model fits the data.

## 2 Double $j$-poles in the triple pomeron ($3I\!\!P$) amplitude

A two-fold, unit intercept pomeron pole is the simplest alternative to the supercritical pomeron. It provides for rising elastic, inelastic and total cross sections, as well as the slope parameter $\sigma^{tot} \sim \sigma^{el} \sim \sigma^{inel} \sim B(s,0) \sim \ln s$ without violating unitarity bounds (higher multiplicity poles with a linear near $t=0$ trajectories, e.g. a tripole, are in conflict with unitarity). Various properties of the dipole pomeron (D$I\!\!P$) as well as their applications to various elastic scattering processes and total cross sections can be found in [8] and references therein.

Less explored are the generalizations of the dipole pomeron to multiparticle reactions. The first question is: how to write correctly a multiparticle amplitude with double poles? To answer it, we first remind that a partial-wave elastic scattering amplitude with a double pomeron pole can be written in the $(j,t)$-representation as

$$Im\, a(j,t) = \frac{\beta^2(j,t)}{[j-\alpha_{I\!\!P}(t)]^2}.$$

The amplitude in the $(s,t)$-representation at large $s$ can be obtained by using a Mellin transform

$$A^{I\!\!D}(s,t) = \frac{1}{2\pi i}\int\limits_{C-i\infty}^{C+i\infty} dj e^{\xi(j-1)}\eta(j)\frac{\beta^2(j,t)}{[j-\alpha_{I\!\!P}(t)]^2} =$$

$$\frac{1}{2\pi i}\frac{d}{d\alpha_{I\!\!P}(t)}\int\limits_{C-i\infty}^{C+i\infty} dj\, e^{\xi(j-1)}\eta(j)\frac{\beta^2(j,t)}{j-\alpha_{I\!\!P}(t)} = \frac{d}{d\alpha_{I\!\!P}(t)}A^{I\!\!P}(s,t),$$

where $\xi = \ln(s/s_0)$, $s_0 = const$ and $\eta(j)$ is the signature factor. The definition of the amplitude $A^{I\!\!P}(s,t)$ corresponding to a simple $j$-plane pole is standard. For not too large $|t|$ the amplitude $A^{I\!\!D}(s,t)$ can be written in the form

$$A^{I\!\!D}(s,t) = i\Big[G(t)\ln(-is/s_0) + \tilde{G}(t)\Big]\Big(-is/s_0\Big)^{\alpha_{I\!\!P}(t)-1}, \tag{2}$$

where we have substituted

$$\eta(j) = -\frac{1+e^{-i\pi j}}{\sin(\pi j)} = i\frac{1}{\sin(\pi j/2)}(-i)^{j-1},$$

and the $1/sin(\pi j/2)$ factor has been absorbed by $G(t)$ and $\tilde{G}(t)$. By using the same procedure, one can write the six-point amplitude with double Pomeron poles in all $t$-channels (the definition of $t_i$ is evident from Fig.2)

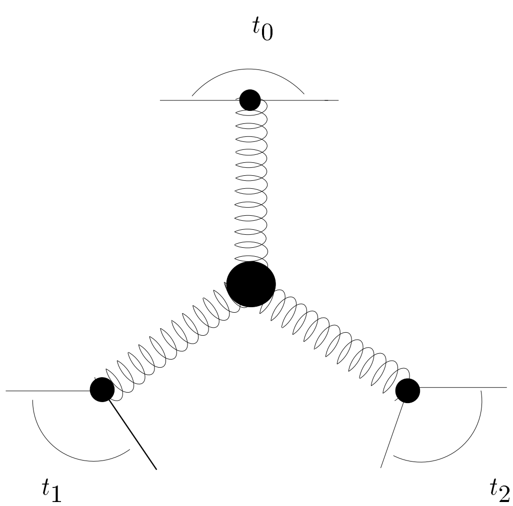


Fig.2. $3I\!\!P$-diagram: the variables $t_i$.

$$A_6^{I\!\!D}(s,M^2,t) = \frac{d}{d\alpha_0}\frac{d}{d\alpha_1}\frac{d}{d\alpha_2}A_6^{I\!\!P}(s,M^2,t_0,t_1,t_2)\mid_{t_0=0,\,t_1=t_2=t;\,\alpha_i=\alpha_{I\!\!P}(t_i)}, \tag{3}$$

where $A_6^{I\!\!P}(s,M^2,t_0,t_1,t_2)$ is the "contribution" of a simple pomeron pole in each of the $t$-channels.

Making use of the generalized optical theorem, we obtain for the single diffractive cross-section (for simplicity we consider all hadrons identical)

$$M^2\frac{d\sigma}{dtdM^2} = \frac{1}{16\pi}\frac{d}{d\alpha_0}\frac{d}{d\alpha_1}\frac{d}{d\alpha_2}\times\beta(t_0,\alpha_0)\beta(t_1,\alpha_1)\beta(t_2,\alpha_2)$$

$$G_{3I\!P}(t_0,t_1,t_2;\alpha_0,\alpha_1,\alpha_2)\left(\frac{s}{M^2}\right)^{\alpha_1+\alpha_2-2}\left(\frac{M^2}{s_0}\right)^{\alpha_0-1}|_{t_0=0,\,t_1=t_2=t;\,\alpha_i=\alpha_{I\!P}(t_i)}\,.$$

Here $G_{3I\!P}(t_0,t_1,t_2;\alpha_0,\alpha_0,\alpha_0)$ is a generalization of the usual triple pomeron, $3I\!P$-vertex. Now we consider this function in more details.

Evidently, $G_{3I\!P}$ can not be constant, since in this case the integrated (over $M^2$ and $t$) cross-section

$$\sigma^{SD}=\int\limits_{\xi_0}^{\xi-\xi_0}d\xi_1\int\limits_{-\infty}^{-|t|_{min}}dt\,\frac{d\sigma}{dtd\xi_1}\propto\xi^3=\ln^3(s/s_0),$$

would violate the unitary inequality $\sigma^{SD}\le\sigma^{tot}$. We remind that in the dipole Pomeron model $\sigma^{tot}\propto\ln(s/s_0)$.

Here and in what follows we use the notation $\xi_1=\ln(s/M^2)$; $\xi_0$ is a large constant restricting the region where Regge behaviour is valid. The upper limit of integration over $t$, $(-|t|_{min})$, generally speaking depends on $M^2/s$, but $|t|_{min}\sim m^2(M^2/s)^2\ll 1$ in the region under consideration ($m$ is the proton mass), so it can be set zero.

Thus, the function $G_{3I\!P}$ should satisfy the following quite general and natural restrictions:

1. Symmetry in $t_1$ and $t_2$;
2. $d\sigma/dtdM^2\neq 0$ at $t=0$;
3. Positivity of the cross-section $d\sigma/dtdM^2$ at any $s,t,M^2$;
4. Unitarity bound $\sigma^{SD}\le\sigma^{tot}$.

It is easy to see that condition 3) can not be satisfied if $G_{3I\!P}$ is a linear function of $t_i$, $\omega_i$ and $t_i$. Below we consider the case of lowest powers in $t_i$ and $\omega_i\equiv\alpha_{I\!P}(t_i)-1$ compatible with all of the imposed conditions on $G_{3I\!P}$, namely

$$G_{3I\!P}(t_0,t_1,t_2;\alpha_0,\alpha_0,\alpha_0)=G^0_{3I\!P}\exp[\bar b(t_0+t_1+t_2)]$$

$$\times[\omega_0+g_1(\omega_1+\omega_2)+g_2\alpha'_{I\!P}(t_1+t_2][\omega_0+\tilde g_1(\omega_1+\omega_2)+\tilde g_2\alpha'_{I\!P}(t_1+t_2)],\qquad(4)$$

where $\omega_i=\alpha_{I\!P}(t_i)-1$ and $\alpha'_{I\!P}$ is the slope of the Pomeron trajectory (inserted to make the parameters $g_2$ and $\tilde g_2$ dimensionless). At large $t_i$, the triple-pomeron vertex $G_{3I\!P}$ may have a complicated dependence on $t_i$, however for the present purposes, here only its small-$t_i$ behaviour will be essential.

One can easily obtain a general expression for the differential cross-section, corresponding to Exp.(4) for $G_{3I\!P}$

$$\frac{d\sigma}{dtd\xi_1}=\frac{1}{16\pi}\beta^3(0)G_{3I\!P}\exp[(B+2\alpha'_{I\!P}\xi_1)t]$$

$$\times\{G_1(\xi-\xi_1)\xi_1^2(2\alpha'_{I\!P}t)^2+G_2(\xi-\xi_1)\xi_1(2\alpha'_{I\!P}t)+G_3\xi_1^2(2\alpha'_{I\!P}t)+G_4\xi_1+G_5(\xi-\xi_1)\},\qquad(5)$$

where

$$G_1=(g_1+g_2)(\tilde g_1+\tilde g_2),\quad G_2=2[g_1(\tilde g_1+\tilde g_2)+\tilde g_1(g_1+g_2)],$$

$$G_3=(g_1+g_2+\tilde g_1+\tilde g_2),\quad G_4=2(g_1+\tilde g_1),\quad G_5=2g_1\tilde g_1.$$

$$B=2b+2\bar b\qquad\text{if}\qquad\beta(t)=\beta(0)e^{bt}.$$

After integration in $t$ and $\xi_1$ we find that the term violating the inequality $\sigma^{SD} \leq \sigma^{tot}$ (it behaves like $\xi \ln \xi$ for $\xi \to \infty$) has a factor $2G_1 - G_2 + G_5$. Hence, by setting

$$2G_1 - G_2 + G_5 = 0, \tag{6}$$

we obtain

$$\sigma^{SD} = C_1 \ln(s/s_0) + C_2 \ln(\ln(s/s_0)) + C_3 + \cdots \tag{7}$$

We did not specify the constants $C_i$ (one can easily express them through $g_i, \tilde{g}_i$) because a more general form for the amplitude can be constructed by taking into account the combination of simple and double pomeron pole. In particular, one can use

$$\frac{d}{d\alpha_i} \quad \to \quad \phi_1(t_i) + \phi_2(t_i)\frac{d}{d\alpha_i}$$

instead of the simple derivative used here with the relevant modification of $\sigma^{SD}$.

To illustrate the applicability of the model and anticipating future detailed fits to the data, here we present only a simple fit to $\sigma^{SD}$ by an approximate choice of the values of the free parameters, without applying the minimization procedure. We give two examples. The first one corresponds to the asymptotic expression (7) with $C_1 = 0.06mb, \quad C_2 = 3mb, \quad C_3 = 0.$ The second curve comes from the expression

$$\sigma^{SD} = C_1 lns + C_2 ln(lns + B) + C_3 + \frac{C_4}{lns + B}, \tag{8}$$

$$C_1 = 0.2mb, \quad C_2 = 2.9mb, \quad C_3 = -1.6mb, \quad C_4 = -12mb, \quad B = 6$$

which takes into account the preasymptotic terms as well. In our opinion, a detailed comparison with the data on the cross-section $d\sigma^{SD}/dtdM^2$ should be made because the model has a quite complicated form of the "$3I\!P$-vertex".

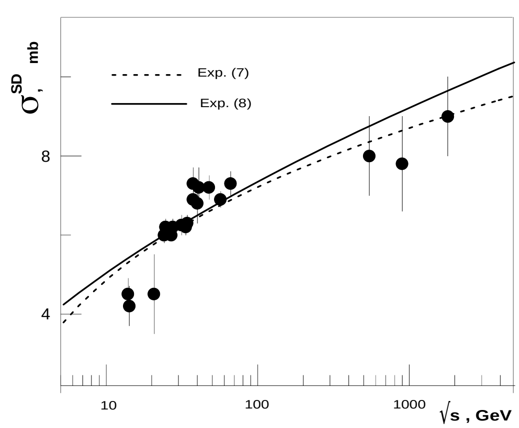


Fig.3. Semi-qualitative fit to the data in the dipole pomeron model presented in this paper. The values of the adjustable parameters were chosen without any minimization procedure - just to illustrate the idea.

# 3 Factorization

Strictly speaking, factorization – in its simplest form – is satisfied only if the relevant exchange (pomeron) is a monom , as in the case of the D-L model [5], in which the scattering

amplitude $\sim s^{\delta}$. Actually, this simplicity is never realized in the nature, since apart form the rising part, diffraction (=pomeron exchange) unavoidably contains also a constant component, required e.g. by duality and/or experimental data at the present, not asymptotical energies.

Factorization is efficient as a crude approximation to reality, but in any quantitative analysis of the available data one should account for the departure from factorization. The problems is technical, rather than conceptual, but it may become crucial for a correct analysis of the data.

The dipole pomeron model always implies the presence of at least two terms in the amplitude, one corresponding to a simple pole exchange (constant cross sections), the other one – to a dipole (logarithmically rising term). To illustrate the modified factorization form, let us write a simplified example of the elastic scattering

$$A^{I\!D}(s,t) = \frac{d}{d\alpha}\beta(t)I\!P(s,t)\beta(t) =$$

$$\beta(t)I\!D(s,t)\beta(t) + \beta'(t)I\!P(s,t)\beta(t) + \beta(t)I\!P(s,t)\beta'(t),$$

where

$$I\!D(s,t) = \ln(-is/s_0)I\!P(s,t), \qquad I\!P(s,t) = (-is/s_0)^{\alpha_{I\!P}(t)-1}.$$

Exact factorization is restored at "asymptotic" energies, when the second and third terms (simple pole contributions) can be neglected. When does it happens – depends on the actual values of the fitted parameters.

The same problem – but in a more complicated form – appears in the amplitude of the DD in a dipole pomeron model. The "generating" amplitude $A_6^{I\!P}$ has the factorized form but the genuine amplitude $A_6^{I\!D}$ is represented by a sum of a few terms and does not factorized. Nevertheless factorization is restored at a far asymptotics and at $t \neq 0$, when the leading term with $(\xi - \xi_1)\xi_1^2$ (see Exp.(4)) dominates.

An important conclusion following from the above arguments is that the "pomeron flux" can be defined only in the asymptotic sense. It may be that the factorized form (1) of the pure hadronic cross-section as well as an analogous form for diffractive DIS are only approximate at available energies. So we must be careful about the conclusions on the quark-gluon content of the pomeron relying too much on the factorization and the concept of the pomeron flux.

The diffractive structure function $F_2^{4(D)}$ is usually calculated making use of the Pomeron structure function $F_2^{I\!P}$, namely

$$\frac{d\sigma^{DDIS}}{dt\,dx_{I\!P}\,dx\,dQ^2} = \frac{4\pi\alpha^2}{xQ^4}(1 - y + \frac{y^2}{2})F_2^{4(D)}, \tag{9}$$

$$F_2^{4(D)} = F_2^{I\!P}(x, Q^2, x_{I\!P}, t) f_{I\!P/p}(x_{I\!P}, t), \tag{10}$$

where the approximation $R = \sigma_L/\sigma_T = 0$ is implied for simplicity. However as we discussed above, the "pomeron flux" makes no sense in the non-asymptotic region. How can $F_2^{4(D)}$ be calculated in this case? We propose a method, based on the idea shown in Fig.4, and briefly described below.

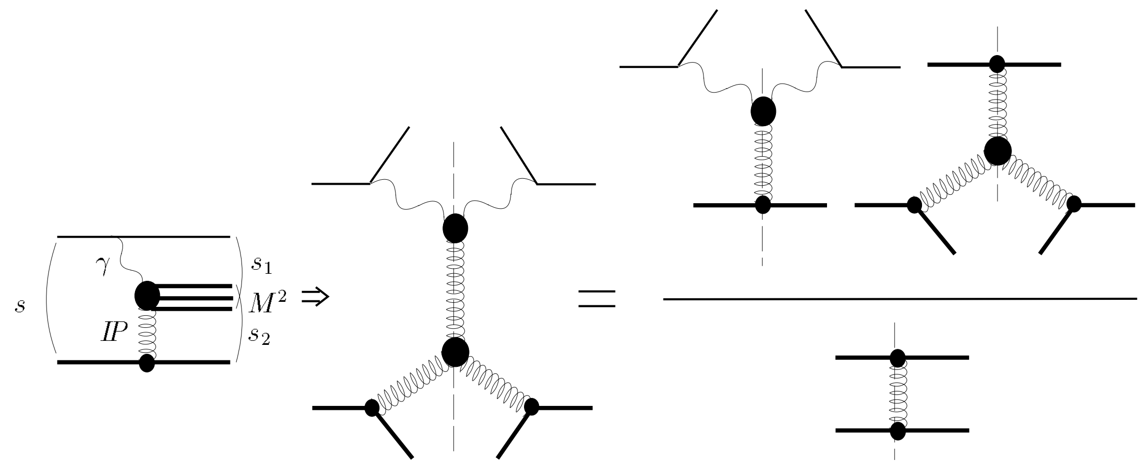


Fig.4. Illustration of the factorization idea for a simple pole

Let us consider first the contribution of a single factorizable simple pole in the crossed channel to the undetected particles. In accord with Fig.4 one may write

$$\frac{d\sigma^{DDIS}}{dtdx_{I\!P}dxdQ^2} = \frac{d\sigma^{DIS}/dxdQ^2 \cdot d\sigma^{SD}/dtd\xi'}{\sigma_{pp}^{tot}(s)} \frac{Q^2}{Q^2(1-x)+m^2x},$$

where $\xi' = ln(s/s_1)$ and $s_1$ is defined in Fig.4. Making use of the well known relations between the cross-sections and structure functions (Exp.(9) for $F_2^{4(D)}$ and its analog for $F_2^p$) we can express $F_2^{4(D)}$ through the usual nucleon structure function and pure hadronic cross-sections:

$$F_2^{4(D)} \propto \frac{1}{\sigma_{pp}^{tot}(s)} F_2^p(x,Q^2) \frac{d\sigma^{SD}}{dtd\xi'}.$$

With account for more contributions (e.g. double pole, simple pole, nonleading reggeons etc.) the cross-section and structure function will be rewritten in the following form

$$\frac{d\sigma^{DDIS}}{dtdx_{I\!P}dxdQ^2} = \frac{Q^2}{Q^2(1-x)+m^2x} \sum_i \frac{d\sigma_i^{DIS}/dxdQ^2 \cdot d\sigma_i^{SD}/dtd\xi'}{\sigma_i^{tot}(s)},$$

$$F_2^{4(D)} \propto \sum_i \frac{1}{\sigma_i^{tot}(s)} F_{2,i}^p(x,Q^2) \frac{d\sigma_i^{SD}}{dtd\xi'},$$

where each of the factors in the sums is the partial contribution to the corresponding quantity

$$\sigma^{tot} = \sum_i \sigma_i^{tot}, \quad F_2^p = \sum_i F_{2,i}^p, \quad \frac{d\sigma^{SD}}{dtd\xi'} = \sum_i \frac{d\sigma_i^{SD}}{dtd\xi'}.$$

# 4 Conclusions

In this paper we have presented a model for the pomeron (dipole pomeron) compatible with unitarity and the experimental data. We have treated only the simplest case of single diffraction, but results are encouraging and we believe that the inclusion of more complicated diagrams, like double DD will complete this study and resolve the puzzle of the so-called decoupling theorems.

Breakdown of factorization is an essential consequence of this approach. Let us stress that the breakdown (or restoration) of factorization in the present model depends on a non-trivial interplay of the $s-$ and $t-$ dependence. We remind that factorizability of the pomeron contribution was a crucial point in most of the calculation and measurements of diffractive deep inelastic scattering (DIS). Moreover, in the Ingelmann-Schlein model for diffractive DIS [3], factorization is an unavoidable ingredient to make the definition of the pomeron structure function, multiplied by the pomeron flux (Exps. (9) and (10)), sensible. The introduction of a realistic pomeron model, unavoidably will modify the simple factorization of the amplitudes and cross-sections. Finally, we note the second important source of non-factorizability, coming from the contribution of secondary (e.g., $f$) trajectories. Their effect will be treated elsewhere. To conclude, let us notice that nonfactorizability is observed also experimentally in diffractive DIS [11] and has become a subject of intensive exploration – both theoretical and experimental.

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